Chung Shan Medical Journal 2014; 25:41-55

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Original Article

Backward Histogram Equalization, Backward Histogram Specification, and Other Backward Variants

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Histogram equalization (HE), a simple and effective algorithm to enhance the contrast of an image, is used in a wide range of image processing applications. Traditionally, it is implemented in the forward direction of the histogram of an image, i.e., from the lowest to the highest gray level. This pattern of thinking is also inherited by other histogram-related techniques. However, it limits possible uses for histograms. In this paper, the backward-operating concept is utilized to propose algorithms for backward histogram equalization (BHE), backward histogram specification, and their extension to histogram redistribution. Our experimental results showed that methods of forward and backward histogram-related equalization can complement each other and that choosing properly between the two can help users obtain more agreeable contrast effects and better brightness preservation.

Key words: Histogram equalization, histogram specification, contrast enhancement, backward histogram equalization, backward histogram specification

Introduction

Contrast enhancement is widely used in various image processing applications[1-3]. Its primary objective is to increase contrast so that one can discern details in an image, making it easier for doctors to easily spot abnormalities. However, excessive enhancement will damage visual effects of original images and even blur them. Therefore, it would be optimal to maintain visual effects while attempting to maximize contrast. To do this, it is necessary to take into account contrast enhancement, brightness preservation, and visual effects.

Histogram equalization (HE) is among the most popular contrast enhancement methods. The goal of HE is to transform the histogram of an input image to a uniform distribution based on the occurrence of gray levels. Theoretically, the transformed image is uniform for continuous histograms[4], but it is almost impossible to have a uniform histogram for digital images. Annoying artifacts are often induced in the output images enhanced by histogram equalization. What is worse, their mean brightness always leans towards the middle of the gray scale of an image regardless of distinctive characteristics of input images. These undesirable effects make viewing monotonic and boring, especially in consumer electronics such as TVs where brightness preservation is usually necessary in order to maintain a wide diversity of visual effects.

To reduce the above-mentioned phenomena, Kim proposed a brightness preserving bi-histogram equalization (BBHE) method[5]. His algorithm

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provides better brightness preservation than HE. Similar to BBHE, Wang et al. proposed dualistic sub-image histogram equalization (DSIHE), which separates an input image into two subimages based on the median of the gray scale of an image, not the mean[6]. Its basic idea is to maximize the entropy of an output image. To verify the effectiveness of their method, the authors illustrated an image with a large portion of gray levels being at 0. They claimed that the quality of the image enhanced by DSIHE was better than BBHE with regard to criteria of mean, average information content (AIC), and background gray level (BGL).

Later, Chen and Ramli proposed that minimum mean brightness error bi-histogram equalization (MMBEBHE)[7] would maximize brightness preservation. They adopted the minimum of the absolute difference between an input mean value and its output one, which they called the absolute mean brightness error (AMBE), as a criterion to compute the threshold gray level in order to separate the input histogram. The threshold gray level is similar in role to the mean value for BBHE or to the median value for DSIHE. Since this algorithm is time consuming, the authors used an approximation method to compute integer values of AMBE recursively. At the same time, Chen and Ramli also proposed another enhancement scheme called recursive mean-separate histogram equalization (RMSHE)[8]. This method can be viewed as an extension of BBHE. First, the mean of the whole histogram is taken as the only threshold gray level. Then, the mean of each subhistogram is taken as the new threshold gray level. This process is repeated r times, and generates 2^r-1 threshold gray levels as well as 2^r subhistograms in total. As the iteration number increases, the mean brightness of the output image approaches that of the original image, but eventually this method will have no effect on contrast enhancement. Though the repeating nature of RMSHE makes the magnitude of brightness preservation adjustable, choosing an appropriate number of iterations remains a challenge.

In addition, in an effort to achieve optimal brightness preservation based on the maximum entropy, Wang and Ye proposed brightness preserving histogram equalization with maximum entropy (BPHEME)[9], which is a method integrating histogram specification (HS) to obtain a specified histogram. This method maximizes the entropy under the constraint of output mean brightness equal to input mean one. BPHEME enhances an input image while preserving the mean brightness, so it is very suitable for consumer electronics such as televisions.

Similar to RMSHE, Sim et al. proposed a method of raising the brightness preservation and enhancing contrast, called recursive sub-image histogram equalization (RSIHE)[10]. Unlike RMSHE, RSIHE uses the median rather than the mean to separate an input histogram iteratively. RSIHE is to DSIHE what RMSHE is to BBHE. RMSHE and RSIHE may generally improve the results enhanced by BBHE and DSIHE, but may also invoke two potential problems, one being how to choose the optimal value of r and the other being the need for subhistograms to have a power of two.

Attempting to address the problems mentioned above, Abdullah-Al-Wadud et al. proposed a type of dynamic histogram equalization (DHE) [11] that would partition an image histogram into subhistograms based on the local minima of the smoothed histogram, and then assign a specified gray level range to each partition before equalizing them separately. DHE, however, does not consider the problem of brightness preservation. Therefore, Ibrahim and Kong proposed a brightness preserving dynamic histogram equalization (BPDHE) method[12]. BPDHE first partitions the image histogram based on the local maxima of the smoothed histogram, rather than the local minima, and then assigns a new dynamic range to each partition. Finally, the output intensity is normalized to make the mean intensity of the resulting image equal to that of the input image.

Around the same period that DHE was developed, Wang and Ward also proposed a straightforward and effective mechanism to control the enhancement process, called weighted thresholded histogram equalization (WTHE)[13]. Their histogram equalization produced show well-enhanced contrast and a very few artifacts.

Arici et al.[14] proposed a general framework

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in which the modified histograms were obtained by minimizing some specific cost functions. They introduced penalty terms to adjust the level of contrast enhancement. Since some modified histograms require matrix multiplication and inversion, the authors finally proposed a low-complexity algorithm, which they called the histogram modification algorithm (HMA), as a means of reducing computational cost and raising visual effects.

Each of these proposed methods plays a very important role in resolving a specific problem, but these contrast enhancement techniques cannot bring about satisfactory quality for a broad range of low-contrast images, especially for those images with histogram components highly concentrated at some pixels. The washed-out appearance, patchiness effects, or/and other artifacts can be easily induced. In an attempt to resolve these problems, Chen et al. proposed gray-level grouping (GLG)[15]. GLG can automatically choose a kind of histogram distribution to optimize image contrast enhancement according to the maximum average distance (AD) between pixels on the grayscale. When applied to some examples like *Phobos* and X-ray image of luggage, the grouping method largely improves the visual effect of the images.

Although GLG can achieve their asserted effects for some cases with a very high probability of the histogram component on the leftmost side of a histogram, it fails in the cases with a very high probability that the histogram component would be on the rightmost side. In order to resolve this problem, Chang and Chang proposed a simple histogram modification scheme to obtain pleasant results[2], a scheme that can be applied to all histogram-related methods using histogram equalization, histogram specification, and histogram redistribution. Moreover, by scrutinizing the implementing procedure of GLG, only an extra constraint imposed on GLG can effectively produce pleasing appearances and preserve as much contrast as possible[16].

Except for traditional one-dimensional histogram equalization, Celik proposed a two-dimensional histogram equalization (2DHE) algorithm to enhance the contrast of an image[17]. However, because it requires a multilevel nested loop, the task is time consuming.

To date, all techniques for contrast enhancement are implemented in the forward direction of a histogram. In this paper, we explore the possibility of implementing histogram-related methods in the backward or reverse direction. Section 2 reviews histogram-based methods. The backward histogram-based methods are proposed in Section 3. Experimental results and discussions are then presented in Section 4. Finally, Section 5 concludes this paper.

Histogram-Based Methods

An image may be considered to have a two-dimensional function, $f(x,y)$ where x and y are spatial (plane) coordinates, and the amplitude of f at any pair of coordinates (x,y) is called the intensity or gray level of the image at that point. When x, y, and f are all finite discrete quantities, we call the image a digital image.

The histogram of a digital image with intensity levels in the range $[0,L–1]$ is a discrete function $h(r_k) = n_k$, where r_k is the kth intensity value and n_k is the number of pixels in the image with intensity r_k. It is common practice to normalize a histogram by dividing its each component by the total number of pixels in the image, denoted by the product RC, where R and C are the row and column numbers of the image. When $N = RC$, a normalized histogram is given by $p(r_k) = n_k/N$, for $k = 0, 1, ..., L–1$. Loosely speaking, $p(r_k)$ is an estimate of the probability of occurrence of intensity level r_k in an image. The sum of all components of a normalized histogram is equal to 1. Thus, it may be called the probability density function (PDF) of an image.

Histogram Equalization

As mentioned earlier, the probability of occurrence of intensity level r_k in a digital image is calculated by

$$p(r_k) = n_k/N,\ k = 0, 1, ..., L–1 \qquad (1)$$

Without loss of generality, r_k is simplified as k in this paper. Therefore, the cumulative distribution function (CDF) of the image is then obtained by

$$c(k)=\sum_{i=0}^{k}p(k),\ k=0,\ 1,\ ...,\ L\text{–}1 \quad (2)$$

Forward histogram equalization, or simply histogram equalization, will map an input gray level k into an output gray level $s_k = T(k)$ using the following transformation or mapping function:

$$s_k = T(k) = (L-1)c(k) \quad (3)$$

Thus, a processed (output) image is obtained by mapping each pixel in the input image with intensity k into a corresponding pixel with level s_k in the output image using (3).

It is obvious from (3) that the mapping function is a scaled version of CDF. The method is referred to as the global or traditional histogram equalization, simply called histogram equalization (HE). In other words, HE uses the histogram of an input image to obtain the mapping function. As will be seen, other histogram-based methods obtain their mapping functions via modified histograms. In (3), the increment at the output gray level $T(k)$ is as follows:

$$\Delta T(k) = T(k) - T(k-1) = (L-1)\, p(k) \quad (4)$$

From (4), it can be seen that the increment of gray level $T(k)$ is proportional to the probability of its corresponding intensity r_k in an input image. The probability of the continuous version of the mapping function $T(k)$ can be proved to be a uniform probability density function[4]. However, the enhanced or output image processed by the corresponding discrete version of HE will not often be uniform. Using the mapping function of discrete HE, the histogram of an output image is made as close to a uniform distribution as possible. Therefore, it is inevitable that HE will invoke undesirable such phenomena as washed-out appearance, patchiness effects, or/and other artifacts.

Brightness Preserving Bi-Histogram Equalization

In order to overcome these problems just mentioned, several methods have been proposed for certain purposes. For example, BBHE was proposed for preserving the mean brightness of a given image while enhancing the contrast in which the mean of an image is denoted by μ, which can be calculated as

$$\mu=\frac{1}{N}\sum_{k=0}^{L-1}kn_k=\sum_{k=0}^{L-1}kp(k) \quad (5)$$

supposing $m = \lfloor \mu \rfloor \in \{0,\ 1,\ ...,\ L\}$. This is called the threshold intensity or gray level. Using this threshold value, the input image is decomposed into two subimages f_L and f_U as follows:

$$f = f_L \cup f_U, \quad (6)$$

where

$$f_L = \{ f(x, y) \mid f(x, y) \le m,\ \forall f(x, y) \in f \}, \quad (7)$$

and

$$f_U = \{ f(x, y) \mid f(x, y) > m,\ \forall f(x, y) \in f \}. \quad (8)$$

Therefore, the subimage f_L consists of intensities $\{0, 1, ..., m\}$, whereas the subimage f_U consists of intensities $\{m+1, m+2, ..., L-1\}$. Under these structures, the PDF of the subimages f_L and f_U can be obtained by

$$p_L(k) = n_k / N_L,\ k = 0,\ 1,\ ...,\ m, \quad (9)$$

and

$$p_U(k) = n_k / N_U,\ k = m+1,\ m+2,\ ...,\ L-1, \quad (10)$$

where N_L and N_U denote the number of pixels in the subimages f_L and f_U, respectively, i.e.,

$$N_L=\sum_{k=0}^{m}n_k, \quad (11)$$

$$N_U=\sum_{k=m+1}^{L-1}n_k, \quad (12)$$

and $N = N_L + N_U$. The cumulative density functions of f_L and f_U are obtained by

$$c_L(k)=\sum_{i=0}^{k}p_L(i),\ k=0,\ 1,\ ...,\ m, \quad (13)$$

and

$$c_U(k)=\sum_{i=m+1}^{k}p_U(i),\ k=m+1,\ m+2,\ ...,\ L-1. \quad (14)$$

Similar to HE, the mapping functions of the subimages are obtained as follows:

$$T_L(k) = mc_L(k),\ k = 0,\ 1,\ ...,\ m, \quad (15)$$

and

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$$T_U(k) = m+1+(L-m-2)c_U(k),\ k = m+1, m+2, ..., L-1. \quad (16)$$

Based on these two mapping functions, the decomposed subimages are histogram-equalized independently, and then combined into the output image. The overall mapping function can then be obtained by combining (15) and (16) as follows:

$$T(k) = \begin{cases} T_L(k) & k = 0, 1, \ldots, m \\ T_U(k) & k = m+1, m+2, \ldots, L-1. \end{cases} \quad (17)$$

Histogram equalization is directly applied to an input image using the above mapping function. This method is called BBHE. Several related methods were proposed later as the variants of BBHE. The main goal of these methods is to find out an appropriate threshold or some thresholds for certain purposes. Unfortunately, these methods possess a potential problem in the upper and lower boundary values (i.e., the first and last nonzero values) of the support of histogram because of the characteristic of histogram equalization. This problem can be resolved using a simple histogram modification scheme[2].

Histogram Specification

It is well known that HE automatically performs the process of contrast enhancement, but its effects are also difficult to control. The transformed image is always globally and evenly-distributed. However, when we need to transform the histogram into a specific form by enhancing the contrast in a certain range, histogram specification (HS) is adopted[18].

The discrete formulation of HS includes four main steps: equalizing the original histogram, specifying the desired histogram and equalizing the specified histogram, and finally finding the desired value by solving the inverse transformation. Some related notations are illustrated here for clarity. The first step is to equalize the original histogram as follows[4]:

$$s_k = T(k) = (L-1)\sum_{i=0}^{k} p_r(i),\ k = 0, 1, ..., L-1. \quad (18)$$

Then, the desired histogram is specified as $p_z(i)$ and the desired histogram is equalized as follows:

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(i) \quad (19)$$

for a value of q. Finally, the following equation is solved to find an approximate solution, z_q:

$$G(z_q) = s_k \quad (20)$$

The desired value of z_q can be found by solving the inverse transformation:

$$z_q = G^{-1}(s_k),\ k = 0, 1, ..., L-1. \quad (21)$$

This operation can find a suitable value of z for each value of s; it equivalently performs a mapping from s to z.

For convenient comparison with the backward histogram specification proposed in the following section, the histogram-specified procedure is summarized as follows[4]:

1. Compute the histogram $p_r(i)$ of the given image and use (18) to find the histogram equalization transformation. Round the resulting values, s_k, to the integer range $[0, L-1]$.
2. Compute all values of the transformation function G using (19) for $q = 0, 1, ..., L-1$, where $p_z(i)$, $i = 0, 1, ..., L-1$, are the values of the specified histogram. Round the values of G to integers in the range $[0, L-1]$. Store the values of G in a table.
3. For every value of s_k, $k = 0, 1, ..., L-1$, use the stored values of G from Step 2 to find the corresponding values of z_q so that $G(z_q)$ is closest to s_k and store these mappings from s to z. When more than one z_q satisfies the given s_k (i.e., the mapping is not unique), choose the *smallest* value by convention.
4. Form the histogram-specified image by mapping every pixel value, s_k, of this image to the corresponding value, z_q, through the mappings found out in Step 3.

Gray-Level Grouping

Conventional contrast enhancement techniques often fail to obtain satisfying results for a variety of low-contrast images, or they need to manually specify the related parameters for achieving desired results. GLG uses an automatic contrast enhancement method to obtain maximum contrast as soon as possible. The basic GLG procedure has three steps: grouping the histogram components of a low-contrast image into a proper number of bins according to a predefined criterion, then

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redistributing these bins uniformly over the grayscale, and finally ungrouping the gray levels previously grouped[15].

In general, GLG may produce a satisfying result compared to conventional contrast enhancement techniques on the images with a very high probability of the histogram component on the leftmost side. GLG has three objectives: achieving a uniform histogram for discrete histograms in the sense that histogram components are redistributed uniformly over the grayscale, utilizing the grayscale more efficiently, and spreading histogram components over the grayscale in a controllable and/or efficient way.

Histogram Modification Scheme

According to the mechanisms for histogram equalization and specification, an image with a very high probability of the histogram component on the leftmost side of a histogram will result in washed-out appearances after histogram equalization; however, one with a very high probability that the histogram component will be on the rightmost side will display patchiness effects after histogram equalization. Although GLG has considered the former, the latter remains a potential problem. Therefore, a simple histogram modification scheme [2] can be used to resolve this kind of problem. The procedure of the scheme for histogram-based equalization and specification is summarized as follows:

1. Find the *first* value and *last* two values of the support of a histogram.
2. Set the *first* value to *zero* and replace the *last* one with the minimum between the *last* two values.
3. Perform histogram equalization.

As for GLG, the scheme is also true, except for the first value of the support of a histogram being replaced with the *minimum* between the *first* two values, instead of *zero*. Experimental results show that an image with a very high probability of the histogram component on the last value, for example, the negative of the example image *Phobos*, will have a pleasant appearance using this simple histogram modification scheme.

Backward Histogram-Based Methods

Backward Histogram Equalization (BHE)

Traditionally, histogram equalization (HE) is performed in accordance with a forward cumulative distribution function like (2). It is easily understood that equalizing histogram in accordance with a backward operation will give rise to another type of histogram equalization. Of course, the effects performed in a backward operation may be better or worse than in a forward operation. Therefore, we can choose the one with the better effects when implementing HE. A slight computational cost will give us a relatively good improvement.

In the following, the backward cumulative distribution function (BCDF) of an image is obtained. Similar to HE, the BCDF is first obtained by

$$c^b(k)=\sum_{i=L-1-k}^{L-1} p(i),\ k=0,1,\dots,L-1. \tag{22}$$

Let

$$p^b(k)=p(L-1-k),\ k=0,1,\dots,L-1, \tag{23}$$

i.e., $p^b(k)$, $k = 0, 1, \dots, L-1$ is equal to the result of flipping the $p(k)$, $k = 0, 1, \dots, L-1$ in the left-right direction, then the following equation can be easily verified

$$c^b(k)=\sum_{i=L-1-k}^{L-1} p(i)=\sum_{i=L-1-k}^{L-1} p^b(L-1-i)=\sum_{i=0}^{k} p^b(i). \tag{24}$$

The backward histogram equalization will map an input gray level k into an output gray level $T^b(k)$ using the following backward transformation or mapping function:

$$s_k^b=T^b(k)=(L-1)\left(1-c^b(L-1-k)\right),\ k=0,1,\dots,L-1. \tag{25}$$

Substituting (24) into (25), we have

$$s_k^b=T^b(k)=(L-1)c(k-1),\ k=0,1,\dots,L-1, \tag{26}$$

where $c(-1) = 0$. When (26) is compared to (3), it is interesting to find that the backward transformation function is very similar to the forward one with $c(k)$ replaced by $c(k-1)$.

Thus, a processed (output) image is obtained by mapping each pixel in the input image with intensity k into a corresponding pixel with level $T^b(k)$ in the output image, using (25) or (26). The

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transformation (mapping) $T^b(k)$ in this equation is called the backward histogram equalization transformation.

Backward Brightness Preserving Bi-Histogram Equalization

Similar to the BHE and BBHE, two related notations of PDF are defined as follows:

$$p_L^b(k) = p_L(m-k),\ k = 0, 1, ..., m \tag{27}$$

and

$$p_U^b(k) = p_U(L-k+m),\ k = m+1, m+2, ..., L-1. \tag{28}$$

Then, two related CDFs are obtained:

$$c_L^b(k) = \sum_{i=m-k}^{m} p_L(i),\ k = 0, 1, ..., m \tag{29}$$

and

$$c_U^b(k) = \sum_{i=L-k+m}^{L-1} p_U(i),\ k = m+1, m+2, ..., L-1. \tag{30}$$

Similar to BBHE, the mapping functions of the subimages are obtained as follows:

$$T_L^b(k) = m\left(1 - c_L^b(m-k)\right),\ k = 0, 1, ..., m \tag{31}$$

and

$$T_U^b(k) = m + 1 + (L-m-2)\left(1 - c_U^b(L-k+m)\right),\quad k = m+1, m+2, ..., L-1. \tag{32}$$

It is easily verified that

$$T_L^b(k) = mc_L(k-1),\ k = 0, 1, ..., m \tag{33}$$

and

$$T_U^b(k) = m + 1 + (L-m-2)c_U(k-1),\quad k = m+1, m+2, ..., L-1. \tag{34}$$

where $c_L(-1) = 0$ and $c_U(m) = 0$. The overall mapping function is obtained by either combining (31) with (32) or combining (33) with (34) as follows:

$$T^b(k) = \begin{cases} T_L^b(k) & k = 0, 1, ..., m \\ T_U^b(k) & k = m+1, m+2, ..., L-1. \end{cases} \tag{35}$$

Thus, a processed (output) image is obtained by mapping each pixel in the input image with intensity k into a corresponding pixel with level $T^b(k)$ in the output image using (35). The transformation (mapping) $T^b(k)$ in this equation is called the backward brightness preserving Bi-histogram equalization (BBBHE) transformation. The same lines can be extended to a variety of forward methods, such as DSIHE, RMSHE, RSIHE, WTHE, and BPDHE.

Backward Histogram Specification

Following BHE and HS, the notations for operation are defined as follows:

$$c_r^b(k) = \sum_{i=0}^{k} p_r^b(i),\ k = 0, 1, ..., L-1 \tag{36}$$

$$c_z^b(k) = \sum_{i=0}^{k} p_z^b(i),\ k = 0, 1, ..., L-1 \tag{37}$$

$$s_k^b = T^b(k) = (L-1)\left(1 - c_r^b(L-1-k)\right),\ k = 0, 1, ..., L-1 \tag{38}$$

$$G^b(z_q^b) = (L-1)\left(1 - c_z^b(L-1-q)\right),\ q = 0, 1, ..., L-1 \tag{39}$$

$$G^b(z_q^b) = s_k^b,\ k = 0, 1, ..., L-1 \tag{40}$$

$$z_q^b = (G^b)^{-1}(s_k^b),\ k = 0, 1, ..., L-1 \tag{41}$$

Equations (38) and (39) are easily to verify as

$$s_k^b = T^b(k) = (L-1)c_r(k-1),\ k = 0, 1, ..., L-1, \tag{42}$$

$$G^b(z_q^b) = (L-1)c_z(q-1),\ q = 0, 1, ..., L-1. \tag{43}$$

As in the traditional or forward histogram specification case, the backward histogram-specified procedure may be summarized for comparison as follows:

1. Compute the histogram $p_r^b(i)$ of the given image and the cumulative histogram $c_r^b(k)$ using (36), and find the histogram equalization transformation according to (38) or (42). Round the resulting values, s_k^b, to the integer range $[0, L-1]$.
2. Compute the cumulative histogram $c_z^b(k)$ of the specified histogram using (37) and find all values of the transformation function G^b using (39) or (43) for $q = 0, 1, ..., L-1$, where $p_z^b(i)$ are the flipping values of the specified histogram, $p_z(i)$. Round the values of G^b to integers in the range $[0, L-1]$. Store the values of G^b in a table.
3. For every value of s_k^b, $k = 0, 1, ..., L-1$, use the stored values of G^b from Step 2 to find the corresponding z_q^b so that $G(z_q^b)$ is closest to s_k^b and store these mappings from s^b to z^b. When more than one value of z_q^b satisfies the given s_k^b (i.e., the mapping is not unique), choose the

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largest value by convention.

4. Form the histogram-specified image by mapping every pixel value, s_k^b, of this image to the corresponding value, z_q^b, through the mappings found in Step 3.

Backward Gray-Level Grouping

As previously mentioned, GLG may generally produce a satisfying result on the images with a very high probability that the histogram component will be on the leftmost side, but it will lose efficacy on the images with a very high probability that the histogram component will be on the rightmost side. GLG has considered some potential risks, but it is limited to an entrenched mind, i.e., a forward logic thinking pattern, so that it cannot keep its edge in some images. Although the contrast enhanced by GLG is still the maximum degree according to the selected criterion, it can lose some visual effects. However, as long as we change the traditional/forward direction of implementation into the reverse/backward one, an astonishing result will be obtained.

Gray-level grouping implemented in the backward direction is considerably intuitive. The steps in the flow chart of the optimized gray-level grouping algorithm[15] need two minor modifications—the histogram of an input image in the first step and the optimal gray-level transformation function in the last step. The original histogram of an input image, $H_n(k)$, $k = 0, 1, ..., M - 1$, needs to be modified as $H_n(M - 1 - k)$, $k = 0, 1, ..., M - 1$. Finally, the optimal gray-level transformation function, $T_{i_{opt}}(k)$, $k = 0, 1, ..., M - 1$, need to be modified as $T_{i_{opt}}(M - 1 - k)$, $k = 0, 1, ..., M - 1$. Note that $H_n(k)$ is equivalent to $n(k)$ in this paper, and M to L.

Backward Histogram Modification Scheme

Since an implementing direction is reversed, the histogram modification scheme also needs to be changed. For clarity, the procedure is summarized as follows:

1. Find the *last* value and *first* two values of the support of a histogram.
2. Set the *last* value to *zero* and replace the *first* one with the minimum between the *first* two values.
3. Perform backward histogram equalization.

Similar to the case in the histogram modification scheme, which is also true for GLG, the last value of the support of a histogram is replaced with the *minimum* between the *last* two values, instead of *zero*. Experimental results show that an image with a very high probability of the histogram component on the first value will have a crisp and pleasant appearance.

Experimental Results and Discussion

In this section, we illustrate two versions of one example image, *Phobos*, the original and negative, because the image has a very high probability that

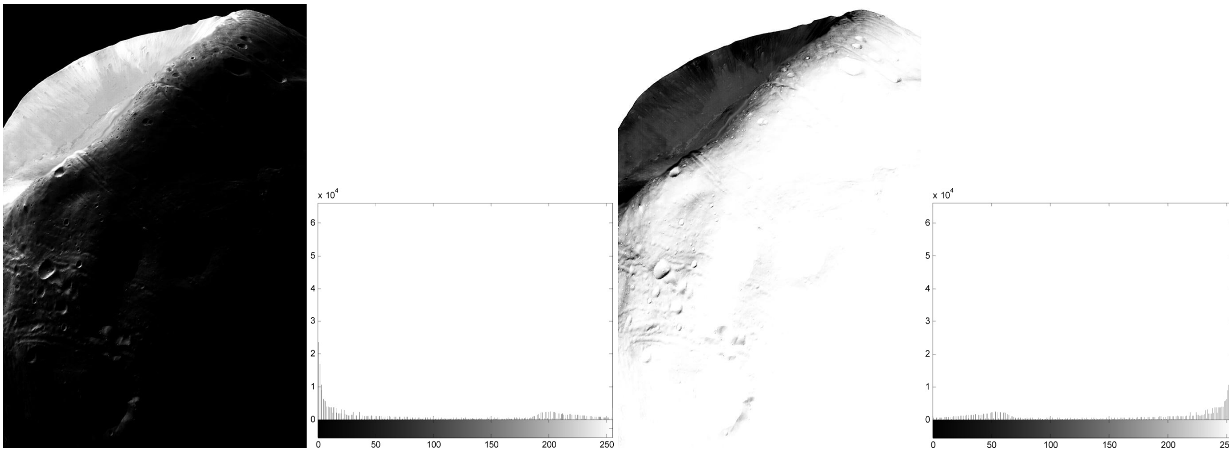

Fig. 1. Image *Phobos* from left to right: original image, histogram, the negative of *Phobos*, and the corresponding histogram.

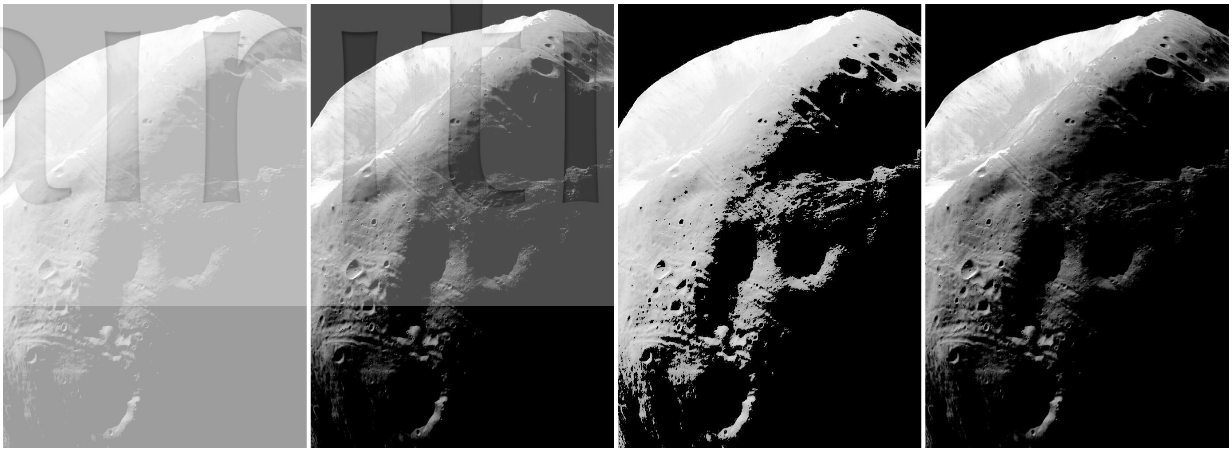

Fig. 2. HE on image *Phobos* from left to right: forward, forward*, backward, and backward*, where * denotes the method using a simple histogram modification scheme.

the histogram component will be on the leftmost side. The images and their histograms are displayed in Fig. 1. Each version was implemented using four methods: HE, BBHE, BPHEME, and GLG. The first two methods apply to histogram equalization; BPHEME to histogram specification, GLG is an automatic redistribution technique. Each method was implemented using forward/traditional and backward directions, and then a simple histogram modification scheme was performed on both, hence four cases. Their transformed results are displayed in Figs. 2 to 9.

In order to quantitatively compare the differences among four cases representing each method, five criteria were adopted: mean, AIC[6, 19], BGL[6], AD[15, 20], and standard deviation (SD) [21]. Chang et al.[20] further proposed an integration of some criteria to assess image quality based on a consistent grey relational grade. The first measure is to calculate the mean brightness of an image. For some consumer electronics such as TVs, it is better to be close to the mean of the original image. The second one is to calculate the average information content of an image. The bigger the value, the larger the information. The third one is to find the background gray level of an image. The enhanced image should be kept as close to the original image as possible in order to avoid a dramatic change in vision. The fourth one is to compute the average distance of an image. In general, the larger the AD value, the better its contrast. The final one is also associated with contrast enhancement. In most situations, the SD value is proportional to the AD one. All related values are listed in Tables I and II.

Fig. 2 shows four enhanced images of *Phobos*. They were produced using forward cumulative probability, forward cumulative probability under a simple histogram modification scheme, the backward one, and the backward one under a simple histogram modification scheme. Results show that the forward one produces washed-out

Table 1. Comparison of methods on *Phobos*.

	Mean	AIC	BGL	PD	SD
Image	31.73	3.51	0	26.70	69.32
HE-*f*	176.23	3.20	157	14.16	29.63
HE-*b*	78.45	3.20	0	50.92	101.33
HE-*f**	50.34	3.40	0	37.14	77.83
HE-*b**	52.47	3.39	0	38.39	79.81
BBHE-*f*	47.10	3.15	24	19.87	54.27
BBHE-*b*	31.79	3.18	0	25.17	60.56
BBHE-*f**	30.60	3.40	0	25.16	62.92
BBHE-*b**	30.37	3.38	0	24.95	62.37
BPHEME-*f*	41.39	3.11	29	9.90	24.55
BPHEME-*b*	23.74	3.13	0	17.03	36.07
BPHEME-*f**	31.85	3.38	0	25.00	57.55
BPHEME-*b**	31.26	3.36	0	24.43	56.03
GLG-*f*	53.25	3.31	0	39.59	83.83
GLG-*b*	64.12	3.24	0	43.16	86.25
GLG-*f**	52.97	3.29	0	39.39	83.39
GLG-*b**	47.77	3.32	0	36.28	78.52

The *f* denotes *forward*; *b*, *backward*; * the method using a simple histogram modification scheme.

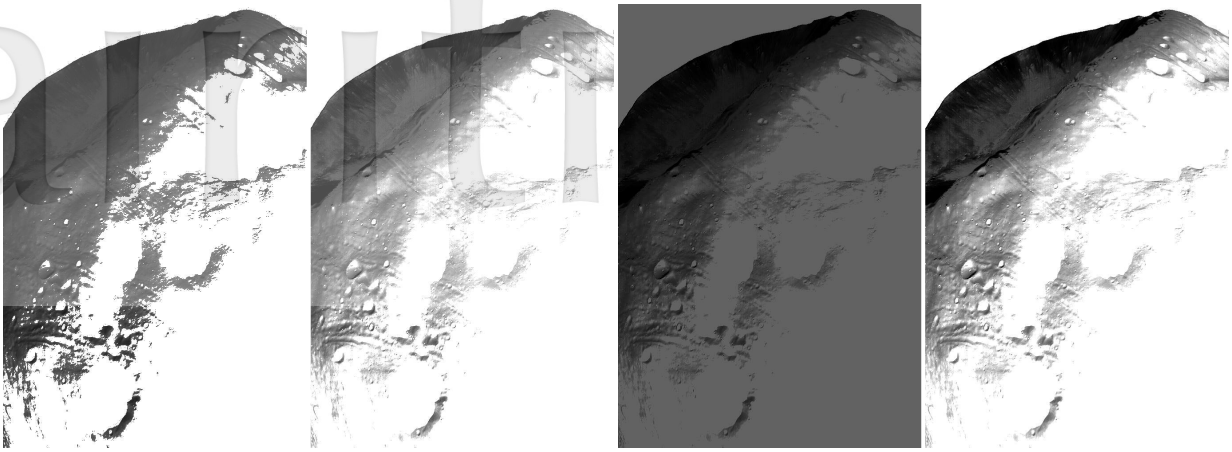

Fig. 3. HE on image the *negative* of *Phobos* from left to right: forward, forward*, backward, and backward*.

appearances. The backward one, however, produces a crisp image with great contrast. In addition, two images with histogram modification are both pleasant and rich in content.

Table 1 provides detailed information about these images. It is obvious from this table that of the four images the backward HE has the maximum PD and SD; the brightness is closer to the original one than the forward one, and its BGL is held in the same place as the original one. The data in this table also shows us that the two images with a simple histogram modification scheme will give us an appropriate contrast enhancement, since the brightness is closer to the original one than two HEs without histogram modification; AIC, larger; but PD and SD, smaller.

Table 2. Comparisons of methods on the negative of *Phobos*.

	Mean	AIC	BGL	PD	SD
image	223.27	3.51	255	26.70	69.32
HE-*f*	176.55	3.20	255	50.92	101.33
HE-*b*	78.77	3.20	98	14.16	29.63
HE-*f**	202.53	3.39	255	38.39	79.81
HE-*b**	204.66	3.40	255	37.14	77.83
BBHE-*f*	223.21	3.18	255	25.17	60.56
BBHE-*b*	207.90	3.15	231	19.87	54.27
BBHE-*f**	224.63	3.38	255	24.95	62.37
BBHE-*b**	224.40	3.40	255	25.16	62.92
BPHEME-*f*	231.26	3.13	255	17.03	36.07
BPHEME-*b*	213.61	3.11	226	9.90	24.55
BPHEME-*f**	223.74	3.36	255	24.43	56.03
BPHEME-*b**	223.15	3.38	255	25.00	57.55
GLG-*f*	190.88	3.24	255	43.16	86.25
GLG-*b*	201.75	3.31	255	39.59	83.83
GLG-*f**	207.23	3.32	255	36.28	78.52
GLG-*b**	202.03	3.29	255	39.39	83.39

The *f* denotes *forward*; *b*, *backward*; * the method using a simple histogram modification scheme.

In general, results from the negative of an image will be contrary to the original image, as shown in Fig. 3 and Table 2.

Figs. 4 and 5 show the results of BBHE, which can be considered as minor adjustments of HE. Table 1 demonstrates that the backward BBHE of *Phobos* is superior to the forward one. However, their corresponding versions under the simple histogram modification show that the forward one is slightly superior to the backward one. Their contrary results, which are obtained from the negative of *Phobos*, can be found in Table 2. In BPHEME, the results are similar to BBHE.

GLG is also is a good example of the reason that we need the pattern of thinking with backward cumulating or grouping. It is obvious from Figs. 8 and 9 that the results using forward grouping is superior to the backward one on the image of *Phobos*, whereas for the case of the negative of *Phobos*, the visual effects obtained by backward grouping surpasses the forward one. As usual, a method with the simple histogram modification may produce more natural and pleasant visual effects than the original one.

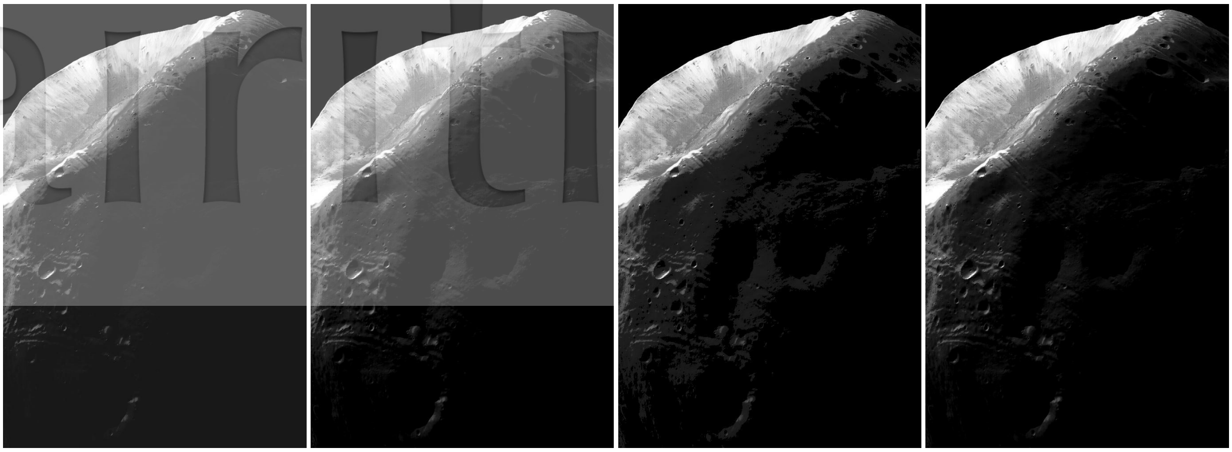

Fig. 4. BBHE on image *Phobos* from left to right: forward, forward*, backward, and backward*.

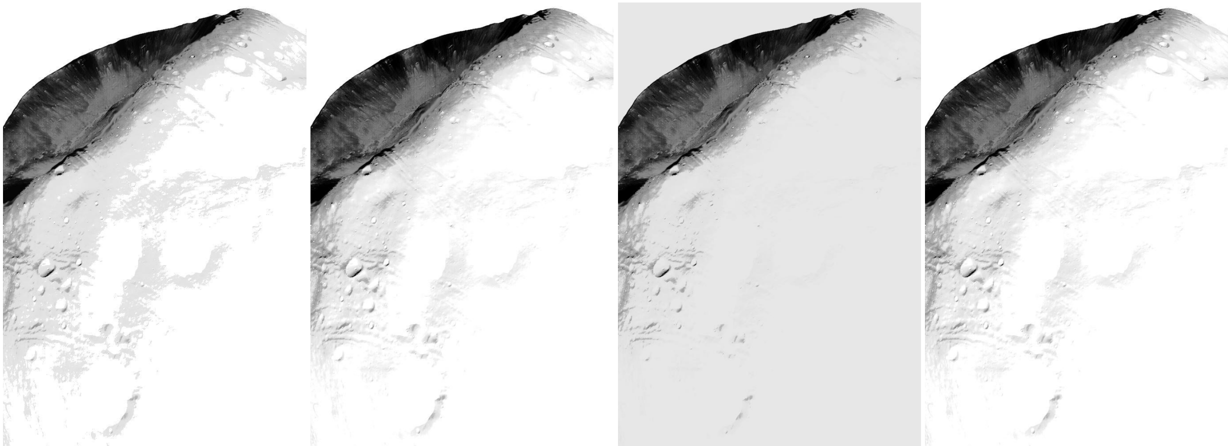

Fig. 5. BBHE on image the *negative* of *Phobos* from left to right: forward, forward*, backward, and backward*.

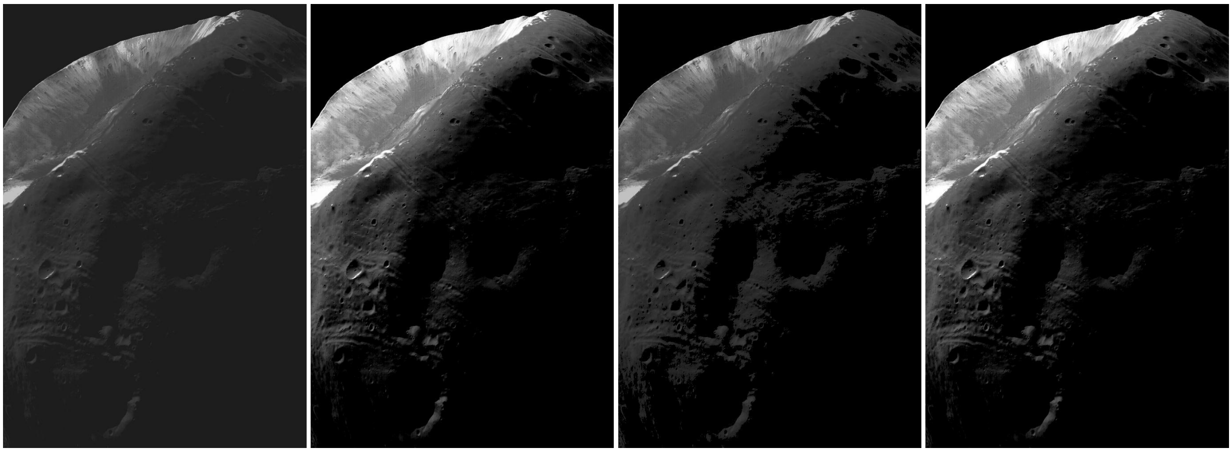

Fig. 6. BPHEME on image *Phobos* from left to right: forward, forward*, backward, and backward*.

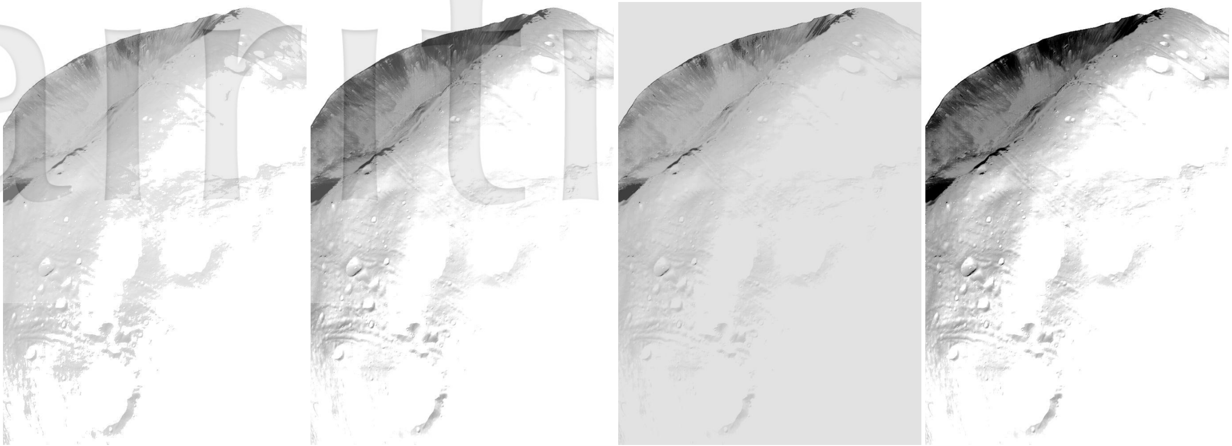

Fig. 7. BPHEME on image the *negative* of *Phobos* from left to right: forward, forward*, backward, and backward*.

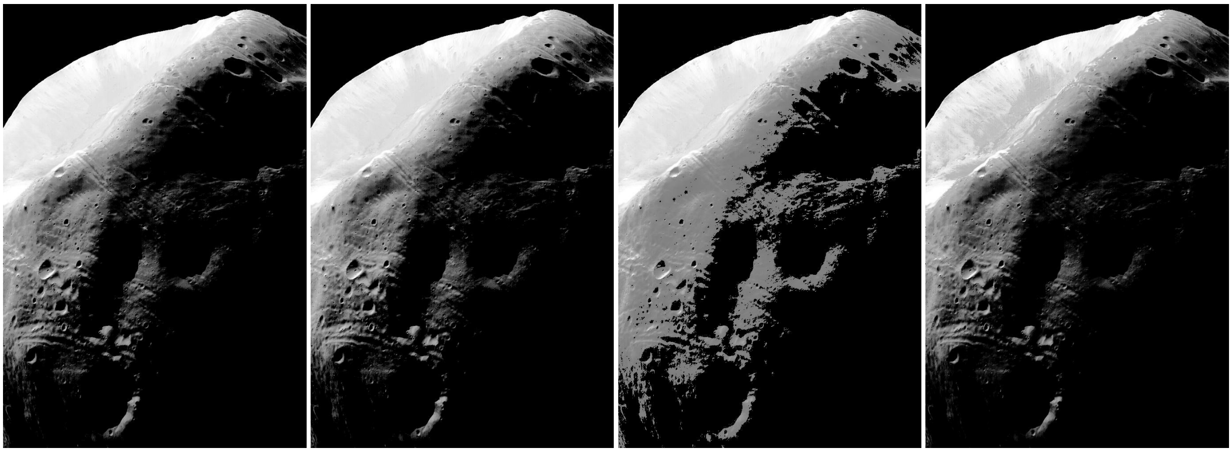

Fig. 8. GLG on image *Phobos* from left to right: forward, forward*, backward, and backward*.

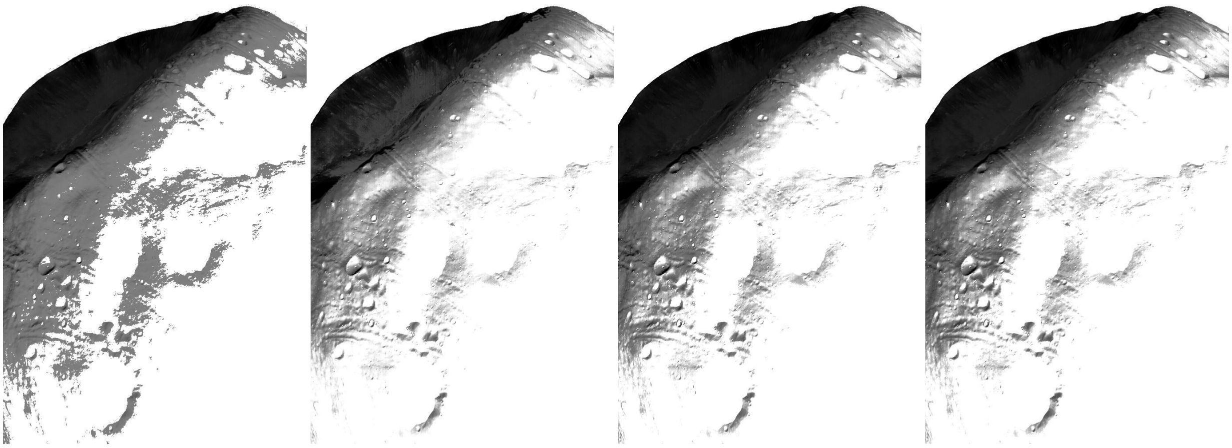

Fig. 9. GLG on image the *negative* of *Phobos* from left to right: forward, forward*, backward, and backward*.

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Conclusions

It is customary for users to equalize the histogram of an image in the forward operation from left to right. To date, existing contrast enhancement methods all follow this operating mode. In general, they can work well for certain objectives, but often overlook some potential problems. Existing histogram, histogram equalization, histogram specification, histogram modification, and histogram redistribution can also be implemented in the backward direction.

This simple histogram modification scheme has been found to be highly effective in improving a potential problem with a large area of gray levels being in the boundary values of the support of a histogram. Thus, a corresponding, i.e., backward, simple histogram modification scheme is developed to provide more pleasant visual effects. Experimental results demonstrate that the proposed concept combined with the traditional operating way can achieve better results.

In the future, the application of backward-operating mode to existing methods will help us effectively solve potential problems we encounter in GLG. In addition, a flexible combination of forward and backward-operating modes for BBHE, DSIHE, etc. is also a promising tool.

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Original Article

後向直方圖等化、後向直方圖指定和其它後向變形

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直方圖等化是一種簡單且有效的增強影像對比的演算法。它廣泛被使用在許多影像處理應用上。傳統上，直方圖等化是從最低灰階到最高的灰階的前向（forward）方向被執行。這種思維模式也被其他直方圖相關的技術所繼承，但是這種思考模式也限制直方圖的可能用途。在本論文中，我們利用後向操作觀念以提出後向直方圖等化，後向直方圖指定的演算法，以及將後向的觀念延伸到直方圖重分布上。我們的實驗結果顯示前向和後向直方圖相關等化的方法可以彼此互補，並且在前後方向上選擇一個適當的途徑有助於使用者獲得更加宜人的對比效果和更佳的亮度保持。

關鍵詞：直方圖等化、直方圖指定、對比增強、後向直方圖等化、後向直方圖指定

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