# 行政院國家科學委員會專題研究計畫 成果報告

## 應用空間資訊技術與混合數值算則於DEM-格網式洪流演算模式之研發
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# 行政院國家科學委員會補助專題研究計畫 ■成果報告 □期中進度報告

## 應用空間資訊技術與混合數值算則
## 於 DEM-格網式洪流演算模式之研發(1)

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計畫參與人員：湯秉宏

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# Kinematic wave computation using an efficient implicit method

Ming-Hseng Tseng


## ABSTRACT

The kinematic wave theorem is the simplest and widely employed distributed routing method used in practical applications for the computation of flood wave propagation in overland and open-channel flows with steep topography. In this paper, an efficient finite-difference implicit MacCormack scheme is developed for the simulation of one-dimensional kinematic wave flows. Simulated results are compared with two analytical solutions and one experimental measurement to assess the performance of an explicit MacCormack scheme, implicit nonlinear kinematic wave scheme and implicit MacCormack scheme. The computed results show that the proposed implicit MacCormack scheme is simple, accurate, highly stable and greatly efficient in solving kinematic wave problems.


**Key words** | channel flow, explicit MacCormack scheme, implicit MacCormack scheme, implicit nonlinear kinematic wave scheme, kinematic wave, overland flow

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## INTRODUCTION

The flow of water through the soil and stream channels of a watershed is a distributed process because the flow rate, velocity and depth vary in the space throughout the watershed. Estimates of the flow rate or water level at important locations in the channel system can be obtained using a distributed flow routing model (Chou *et al.* 1988). Distributed flow routing models can be used to describe the transformation of storm rainfall into runoff over a watershed. This, in turn, can produce a flow hydrograph for the watershed outlet. This hydrograph becomes the input at the upstream end of a river or pipe system routed to the downstream end. For many practical purposes, the spatial variation in velocity across the channel with respect to depth can be ignored. This way, the variation of the flow process can be isolated only in the main direction of the flow along the channel.

Flood wave propagation in overland and channel flows may be described by complete equations of motion for unsteady nonuniform flow, known as dynamic wave equations. This was first proposed by Barre de Saint-Venant in 1871. Due to the fact that dynamic wave equations involve a high degree of complexity in computation, the Saint-Venant equations have been simplified. Several simplified forms are being used to efficiently implement the approximate wave models. Under a different set of simplifying assumptions, all distributed flow routing models can be classified into the kinematic wave, the diffusion wave, the quasi-steady dynamic wave and the full dynamic wave models. The simplest distributed model is the kinematic wave model, which neglects local acceleration, convective acceleration and pressure terms in the momentum equation; that is, it assumes that the friction and gravity forces balance each other. In cases where the bed slope is dominant in the momentum equation, most flood waves behave as kinematic waves. Previous studies (Lighthill & Whitham 1955; Henderson 1966) have proven that the velocity of the main part of a natural flood wave approximates that of a kinematic wave in steep rivers

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(bed slope > 0.002). The one-dimensional kinematic wave model can be used to simulate these overland and open-channel flows (Lighthill & Whitham 1955; Freeze 1978; Cundy & Tento 1985; Chou *et al.* 1988; Singh 1996).

Numerous explicit and implicit finite-difference schemes have been widely used (Cunge *et al.* 1980; Chou *et al.* 1988) to solve different distributed flow routing models. Several advantages of the explicit MacCormack (EMAC) scheme make the method a popular choice in computational hydraulics. Firstly, the scheme is a shock-capturing technique with a second-order accuracy both in time and space. Secondly, the inclusion of the source terms is relatively simple. Thirdly, it is suitable for implementation in an explicit time-marching algorithm. Tseng & Chu (2000) investigated the accuracy of four different versions of the total variation diminishing (TVD) MacCormcak explicit scheme. Tseng & Wang (2004) discovered that the explicit MacCormack model, when compared with the TVD-MacCormack, may be a superior choice for simulating subcritical flows with strong bed topography variations. This was proven by discretizing the source terms at each step with forward or backward difference, in the same manner as the flux gradient. Kazezyılmaz-Alhan *et al.* (2005) studied the reliability of the explicit MacCormack, explicit linear, implicit nonlinear and the four-point implicit schemes for solving one-dimensional overland flows. Under the Courant–Friedrich–Lewy (CFL) stability condition, they concluded that the explicit MacCormack method should be generally preferred as a solution technique over the explicit linear and implicit methods for solving overland flow problems.

This paper proposes a robust implicit MacCormack (IMAC) scheme to be used in the construction of a simple, accurate and efficient solver for kinematic wave computations. The performance of the proposed method will be tested against the traditional EMAC scheme and the widely used implicit nonlinear kinematic wave (INKW) scheme. All these schemes, including the proposed one, will be used to solve overland and channel flow problems. The comparative results will be presented.

None of the previous studies have reported the results of the use of the implicit MacCormack scheme to calculate kinematic wave flows. This paper, therefore, is the first attempt to explore the potential of the proposed method for one-dimensional overland and channel flow routing. In the following sections, the formulations of the proposed approach are defined, the numerical testing scenarios are described, and the stability, accuracy and efficiency of these schemes are compared and evaluated.

## GOVERNING EQUATIONS

Based on the hydrostatic pressure distribution and small channel slope assumptions, an unsteady open-channel flow can be described by Saint-Venant Equations (Chou *et al.* 1988). The shallow water flows can be presented in a conservative form for continuity (Equation (1)) and momentum (Equation (2)) as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(\beta Q^2/A)}{\partial x} + gA\frac{\partial y}{\partial x} = gA(S_o - S_f) + \beta q_l v_x \tag{2}$$

where $t$ represents the time, $x$ is the longitudinal distance along a channel, $A$ is the wetted cross-sectional area, $Q$ is the flow rate volume, $q_l$ is the lateral inflow rate, $\beta$ is the momentum correction factor, $g$ is the gravitational acceleration, $y$ is the water depth, $S_o$ is the bed slope, $S_f$ is the friction slope and $v_x$ is the lateral inflow velocity.

The momentum equation consists of six terms responsible for the physical processes that govern the flow momentum. These terms are the local acceleration term, the convective acceleration term, the pressure force term, the gravity force term, the friction force term and the lateral inflow force term. The local and convective acceleration terms represent the effect of inertial forces on the flow. For a kinematic wave, the acceleration and the pressure terms in the momentum equation are negligible. Therefore, the wave motion can be described principally by the equation of continuity. The kinematic wave model is defined by the following Equations (Chou *et al.* 1988):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{3}$$

$$A = \alpha Q^{\beta} \tag{4}$$

For example, the Manning equation written with $S_o = S_f$ and $R = A/P$ is

$$A = \left(\frac{nP^{2/3}}{c_I S_o^{1/2}}\right)^{3/5} Q^{3/5} \quad (5)$$

where $n$ denotes the Manning roughness coefficient and $P$ represents the wetted perimeter where $c_I$ is 1 for SI units and 1.49 for British units.

By differentiating Equation (5) and substituting it into Equation (3), the result will be

$$\frac{\partial Q}{\partial x} + \alpha\beta Q^{\beta-1}\frac{\partial Q}{\partial t} = q_l \quad (6)$$

The characteristic equations for a kinematic wave can be derived from Equation (6) as

$$\frac{dQ}{dx} = q_l \quad (7)$$

$$c_k = \frac{dx}{dt} = \frac{1}{\alpha\beta Q^{\beta-1}} = \frac{dQ}{dA} \quad (8)$$

where $c_k$ is the kinematic wave celerity. This implies that an observer moving at a velocity of $c_k$ with the flow would see the flow rate increasing at a rate of $q_l$.

## NUMERICAL APPROACHES

The computational domain is discretized as $x_i = i\Delta x$ and $t^j = j\Delta t$, where $\Delta x$ is the size of a uniform mesh and $\Delta t$ is the time increment. The performances of the three finite-difference schemes are investigated in this study. These schemes include the widely used INKW scheme, the traditional EMAC scheme and the proposed IMAC scheme.

### Nonlinear implicit scheme

The implicit nonlinear finite-difference formulation of kinematic wave Equations (Li *et al.* 1975), denoted as the INKW scheme, can be expressed as

$$\begin{cases} f\left(Q_{i+1}^{j+1}\right) = \frac{\Delta t}{\Delta x} Q_{i+1}^{j+1} + \alpha\left(Q_{i+1}^{j+1}\right)^{\beta} - C \\ C = \frac{\Delta t}{\Delta x} Q_i^{j+1} + \alpha\left(Q_{i+1}^{j}\right)^{\beta} + 0.5\Delta t\left(q_{l_{i+1}}^{j+1} + q_{l_{i+1}}^{j}\right) \end{cases} \quad (9)$$

where $f(Q_{i+1}^{j+1})$ represents a nonlinear function of the unknown flow rate $Q_{i+1}^{j+1}$. Equation (9) is solved by the Newton–Raphson iteration method for $Q_{i+1}^{j+1}$ as

$$\left(Q_{i+1}^{j+1}\right)_{k+1} = \left(Q_{i+1}^{j+1}\right)_k - \frac{f\left(Q_{i+1}^{j+1}\right)}{f'\left(Q_{i+1}^{j+1}\right)} \quad (10)$$

The convergence criterion for the iteration process is

$$\left|f\left(Q_{i+1}^{j+1}\right)_{k+1}\right| \le \varepsilon \quad (11)$$

where $f'(Q_{i+1}^{j+1})$ represents the first derivative of $Q_{i+1}^{j+1}$ and $\varepsilon$ is an error criterion.

### Explicit–implicit MacCormack schemes

Based on a predictor–corrector–updating three-step procedure (MacCormack 1969, 1985; Tseng & Wang 2004), the first step follows the algorithm as

$$\Delta A_i^p = -\frac{\Delta t}{\Delta x}(Q_{i+1} - Q_i) + q_{l_i}\Delta t \quad (12)$$

$$\left(1 + \lambda\frac{\Delta t}{\Delta x}\right)\delta A_i^p = \Delta A_i^p + \lambda\frac{\Delta t}{\Delta x}\delta A_{i+1}^p \quad (13)$$

$$A_i^p(t + \Delta t) = A_i(t) + \delta A_i^p \quad (14)$$

The correct step follows the algorithm as

$$\Delta A_i^c = -\frac{\Delta t}{\Delta x}(Q_i - Q_{i-1})^p + q_{l_i}^p\Delta t \quad (15)$$

$$\left(1 + \lambda\frac{\Delta t}{\Delta x}\right)\delta A_i^c = \Delta A_i^c + \lambda\frac{\Delta t}{\Delta x}\delta A_{i-1}^c \quad (16)$$

$$A_i^c(t + \Delta t) = A_i(t) + \delta A_i^c \quad (17)$$

where the subscript $p$ and $c$ stand for the predictor and the corrector steps, respectively. The parameter $\lambda$ is chosen to ensure unconditional linear stability. This requires

$$\lambda = \max\left(0, |c_k| - \frac{\Delta x}{\Delta t}\right) \quad (18)$$

The third step updates the algorithm with the unknown variables of $A$ and $Q$ as follows:

$$A_i(t + \Delta t) = \frac{1}{2}(A_i^p + A_i^c) \quad (19)$$

$$Q_i(t+\Delta t) = \left(\frac{c_I S_o^{1/2}}{nP^{2/3}}\right) A_i^{5/3}(t+\Delta t) \quad (20)$$

In the above equations, the parameter $\lambda$ is used to represent the IMAC and EMAC schemes in the same formulation. The IMAC and EMAC schemes have second-order accuracy in both time and space. The relations are presented as

$$\left.\begin{array}{l} \lambda = 0 \rightarrow \text{EMAC} \\ \lambda \neq 0 \rightarrow \text{IMAC} \end{array}\right\} \quad (21)$$

For the stability of the EMAC scheme, the CFL condition must be satisfied:

$$\text{CFL} = \frac{\Delta t}{\left[\frac{\Delta x}{u+c}\right]} \leq 1.0 \quad (22)$$

where CFL is the Courant number. For the implicit methods such as the IMAC and INKW schemes, the unconditional stability is the consideration.

## NUMERICAL RESULTS AND DISCUSSION

An important test of the quality of a computational model is its ability to reproduce standard test cases or benchmarks. The numerical simulations of overland and open-channel flow benchmark cases are presented. The results are compared with the corresponding analytical solutions in this section. All the simulations were executed on a Toshiba Tablet PC (1.33 GHz CPU, 752 MB RAM).

For the quantitative comparison of the performance characteristics of the numerical schemes, two evaluation indexes are defined by the $L_2^m$ norm and the *Efficiency* index.

The mean relative error is described by the $L_2^m$ norm:

$$L_2^m(\%) = \frac{100}{N}\left[\frac{\sum\left(H_i^{\text{cal}} - H_i^{\text{analytic}}\right)^2}{\sum(H_i^{\text{analytic}})^2}\right]^{1/2} \quad (23)$$

where $N$ is the number of observations, and $H_i^{\text{cal}}$ and $H_i^{\text{analytic}}$ are the simulated and analytical values at grid point ($i$), respectively.

The Efficiency index is defined as

$$\text{Efficiency}(\Delta t = X) = \frac{\text{CPU time of } \Delta t_{\min}}{\text{CPU time of } \Delta t = X} \quad (24)$$

### Overland flow case

The performance of the proposed numerical method was first tested by computing the overland flow hydrograph associated with uniform rainfall on a parking lot. An effective rainfall continues for 1,500 s with an intensity of 0.1 m/h over a 500 m long and 100 m width parking lot. The slope of the plane is 0.01 and the Manning roughness coefficient is 0.005. The initial condition is a dry bed state.

First, it is important to inspect the behavior of a numerical scheme over a range of grid sizes and time

**Table 1** | Sensitivity analysis for test cases using the IMAC scheme

| Test cases | $\Delta x$ (m) | $\Delta t$ (s) | $L_2^m$(%) Depth ($y$) | $L_2^m$(%) Discharge ($Q$) | CPU time (s) |
|---|---|---|---|---|---|
| Overland flow | 25 | 10 | 0.0471 | 0.0781 | 0.3203 |
| Overland flow | 10 | 5 | 0.0144 | 0.0240 | 0.5195 |
| Overland flow | 1 | 1 | $2.26 \times 10^{-3}$ | $3.76 \times 10^{-3}$ | 8.4531 |
| Overland flow | 1 | 0.5 | $3.95 \times 10^{-4}$ | $6.64 \times 10^{-4}$ | 11.937 |
| Channel flow | 1,500 | 50 | 0.0564 | 0.0935 | 0.6289 |
| Channel flow | 300 | 10 | $9.62 \times 10^{-3}$ | 0.0159 | 3.1875 |
| Channel flow | 30 | 2 | $2.81 \times 10^{-3}$ | $4.64 \times 10^{-3}$ | 16.734 |
| Channel flow | 30 | 1 | $2.35 \times 10^{-3}$ | $3.88 \times 10^{-3}$ | 23.305 |

increments. A quantitative comparison of the simulated results using the IMAC scheme can be found in Table 1. The analytical solution for the kinematic wave flow with constant rainfall is obtained by Eagleson's solution (1970). The results indicate that the computational error ($L_2^m$) decreases as the grid size ($\Delta x$) or the time increment ($\Delta t$) decreases. On the other hand, the computational time increases as the grid size or the time increment decreases. These results clearly show that the proposed IMAC scheme is essentially conservative. This comparison also demonstrates that the proposed IMAC scheme can acquire remarkable accuracy with $\Delta x = 1$ m and $\Delta t \leqq 1$ s, where the $L_2^m$ norms of depth and discharge are both less than $5 \times 10^{-3}$%. The time step of 0.5 s leads to a Courant number approximately between 0.9 and 0.5.

Figure 1 compares the simulation results of the water depth, as well as the flow rate, with the analytical solution for the EMAC, IMAC and INKW schemes, respectively. A uniform mesh distribution with 500 cells and a time increment of 0.5 s was used in the computation. As can be seen, the agreement between the analytical solution and the numerical solution is remarkable for both the water depth and the flow rate, except for the INKW scheme. Since the INKW scheme is only a first-order method, significant numerical damping leads to smearing of the peak depth and peak discharge, as shown in the inset graphs. The results of the simulation also indicate that the IMAC and EMAC schemes are slightly better than those of the INKW scheme. Figure 2 demonstrates the simulation result of the water depth using the IMAC scheme with $\Delta t = 1$ s, 10 s, and 100 s, respectively. It can be observed that the simulated hydrograph closely follows the analytical solution, even in the case of a very large time step employed.

As indicated in Table 2, the simulated results of the IMAC and EMAC schemes for the $\Delta x = 1$ m and $\Delta t = 0.5$ s case are identical, except in the aspect of time consumption where the IMAC scheme showed some increase. However, for the cases where $\Delta t \geq 1$ s, the traditional EMAC scheme failed to compute the overland flow. In contrast, the proposed IMAC scheme was able to simulate very well, even in the case where $\Delta t = 100$ s. All the relative errors for the $\Delta t$ (s) values varying from 0.5 to 100 are less than 0.5%,

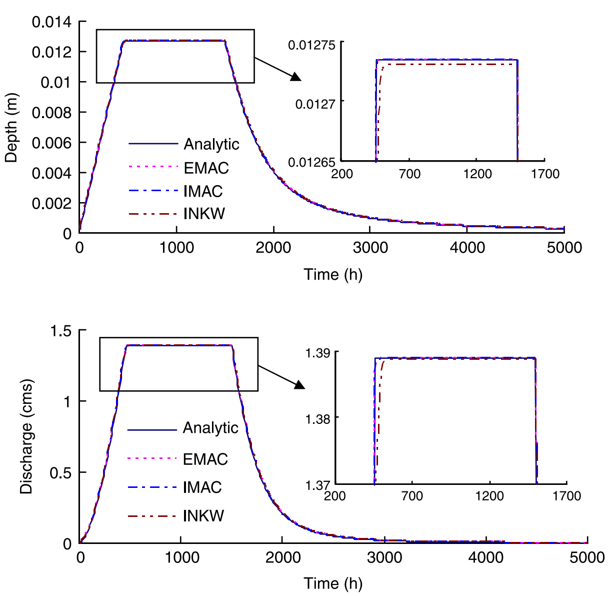


**Figure 1** | Solution of overland flow using the EMAC, IMAC and INKW schemes ($\Delta t = 0.5$ s).

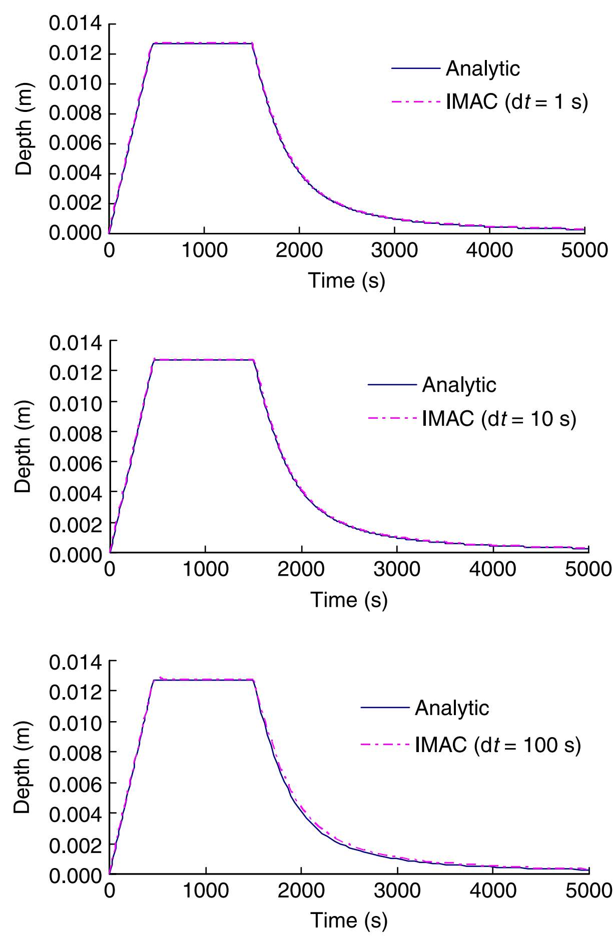


**Figure 2** | Simulation hydrograph of depth for overland flow.

**Table 2** | Simulation results for the overland flow problem

| Schemes | $\Delta x$ (m) | $\Delta t$ (s) | $L_2^m$(%) Depth ($y$) | $L_2^m$(%) Discharge ($Q$) | CPU time (s) | Efficiency |
|---|---|---|---|---|---|---|
| EMAC | 1 | 0.5 | $3.95 \times 10^{-4}$ | $6.64 \times 10^{-4}$ | 9.5117 | 1.00 |
| IMAC | 1 | 0.5 | $3.95 \times 10^{-4}$ | $6.64 \times 10^{-4}$ | 11.937 | 0.80 |
| EMAC | 1 | 1 | N/A | N/A | N/A | N/A |
| IMAC | 1 | 1 | $2.26 \times 10^{-3}$ | $3.76 \times 10^{-3}$ | 8.4531 | 1.13 |
| IMAC | 1 | 5 | $6.26 \times 10^{-3}$ | 0.0104 | 3.2539 | 2.92 |
| IMAC | 1 | 10 | 0.0176 | 0.0293 | 2.1211 | 4.48 |
| IMAC | 1 | 50 | 0.1000 | 0.1652 | 0.7305 | 13.02 |
| IMAC | 1 | 100 | 0.2770 | 0.4516 | 0.4805 | 19.80 |
| INKW | 1 | 0.5 | 0.0458 | 0.0775 | 13.5195 | 0.70 |
| INKW | 1 | 1 | 0.0506 | 0.0855 | 6.9375 | 1.37 |
| INKW | 1 | 5 | 0.0935 | 0.1561 | 1.9539 | 4.87 |
| INKW | 1 | 10 | 0.1484 | 0.2456 | 1.0195 | 9.33 |
| INKW | 1 | 50 | 1.182 | 1.878 | 0.2695 | 35.29 |
| INKW | 1 | 100 | 2.828 | 4.324 | 0.1797 | 52.93 |

*Note*: "N/A" = Not Available.

which demonstrates that the proposed IMAC model is accurate, robust and highly stable.

For the higher time increment values, a linear increase in the value of $\Delta t$ was employed to overcome the difficulty imposed by the initial dry bed condition. The imposed difficulty is apparent when a very large initial time step was used. The CPU times required for the EMAC scheme ($\Delta t = 0.5$ s), the IMAC scheme ($\Delta t = 10$ s) and the IMAC scheme ($\Delta t = 100$ s) are 9.5117 s, 2.1211 s and 0.4805 s, respectively. The $\Delta t$ value is an indicator of the efficiency of the numerical scheme, the value of which increases with the efficiency of the numerical scheme. As shown in Table 2, the values of the *Efficiency* index for the EMAC scheme ($\Delta t = 0.5$ s), the IMAC scheme ($\Delta t = 10$ s) and the IMAC scheme ($\Delta t = 100$ s) are 1.00, 4.48 and 19.80, respectively. The results clearly indicate that the proposed IMAC scheme, which allowed the use of time steps of magnitudes bigger than those in the EMAC scheme, is a very robust algorithm which can be used to calculate the unsteady overland flow.

Table 2 also enumerates the simulation results for the overland flow problem using the INKW scheme. It is apparent that the INKW scheme appears to be more efficient than the IMAC scheme. However, the levels of accuracy are lower in magnitude than those of the IMAC scheme. Since the INKW scheme has only a first-order accuracy, the resulting computation time is slightly less than that of the second-order scheme (IMAC).

## Channel flow case

The above benchmark problem only tests the proposed approach to simulate transient overland flow. In this subsection, we demonstrate the capability of the proposed model to describe an unsteady channel flow. In order to do this, a flood event over a 1500-foot-long, 200-foot-wide rectangular channel with a bed slope of 0.01, and a Manning roughness coefficient of 0.035, was simulated to compute the outflow hydrograph. The initial condition was a uniform flow of 2,000 cfs along the channel. At the upstream boundary, the following hydrograph $Q(t)$ was imposed as

$$\begin{cases} Q(t)=2000\,\text{cfs} & 0\le t\le 12\,\text{min} \\ Q(t)=2000+\frac{1000}{12}(t-12)\,\text{cfs} & 12\le t\le 60\,\text{min} \\ Q(t)=6000-\frac{1000}{12}(t-60)\,\text{cfs} & 60\le t\le 108\,\text{min} \\ Q(t)=2000\,\text{cfs} & 108\le t\le 150\,\text{min} \end{cases} \quad (25)$$

The simulated outflow hydrograph of water depth at the downstream of the channel using the IMAC scheme

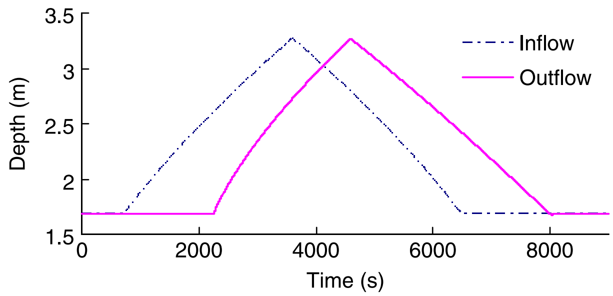


**Figure 3** | Inflow and outflow hydrographs using the IMAC scheme ($\Delta t = 2$ s).

with $\Delta x = 30$ ft and $\Delta t = 2$ s is given in Figure 3. The simulation result indicates that the total areas under the hydrographs are almost identical. In other words, the proposed scheme can conserve the total water mass when the flood wave passes through the channel reach. The simulation results evidently reveal that the kinematic wave

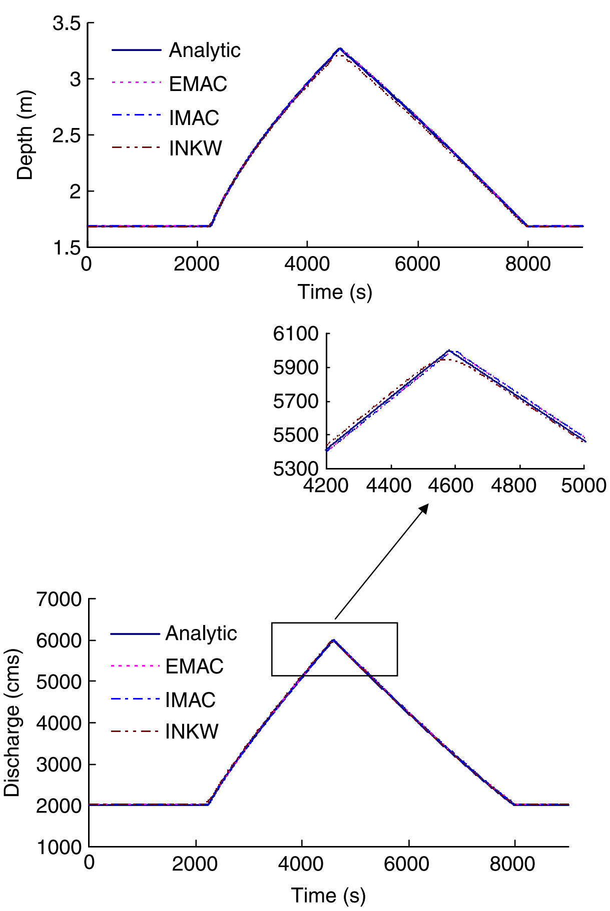


**Figure 4** | Solution of channel flow using the EMAC, IMAC and INKW schemes ($\Delta t = 1$ s).

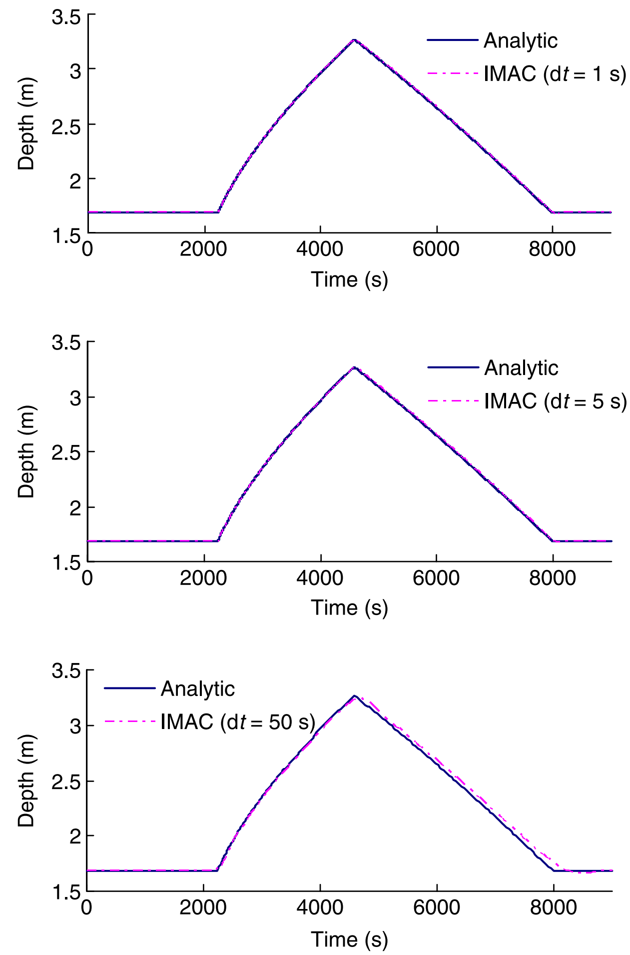


**Figure 5** | Simulation hydrograph of depth for channel flow.

does not subside or disperse, but changes its shape because its velocity depends upon the depth (Chou *et al.* 1988).

Table 1 also shows the behavior of the IMAC scheme over a range of grid sizes and time increments for the channel flow case. The analytical solution for kinematic wave channel flow can be obtained by simultaneously solving the characteristic Equations (7) and (8). These results indicate that the computational error ($L_2^m$) decreases as the grid size ($\Delta x$) or the time increment ($\Delta t$) decreases. On the other hand, the computational time increases as the grid size or the time step decreases. These results clearly verify that the proposed IMAC scheme is conservative. This comparison also evidently demonstrates that the proposed IMAC scheme can acquire remarkable accuracy with $\Delta x = 30$ m and $\Delta t \leqq 2$ s, where both the $L_2^m$ norms of depth and discharge are less than

$5 \times 10^{-3}$%. The time increment of 1 s leads to a Courant number approximately between 0.9 and 0.5.

Figure 4 compares the simulation results of the water depth, as well as the flow rate, with the analytical solution for the EMAC, IMAC and INKW schemes, respectively. A uniform mesh distribution of 500 cells and time step of 1 s was used in the computation. As can be seen, an excellent match between the analytical solution and the numerical solution is notable for the EMAC and IMAC schemes. Since the INKW scheme has only a first-order accuracy, significant numerical damping leads to smoothing of the peak depth and peak discharge, as seen in the inset. The results of simulation also indicate that the IMAC and EMAC schemes are slightly better than the INKW scheme. Figure 5 illustrates the simulation result of the water depth using the IMAC scheme with $\Delta t = 1$ s, 5 s and 50 s, respectively. It can be observed that the simulated hydrograph closely follows the analytical solution even in the case of the very large time step employed. The simulated hydrograph of water depth shows a faster wave movement for the $\Delta t = 50$ s case, but the overall accuracy is still satisfactory.

As indicated in Table 3, the simulated results of the the IMAC scheme are almost identical with the IMAC and EMAC schemes for the $\Delta x = 30$ ft and $\Delta t = 1$ s cases. For the case with $\Delta t \geqq 2$ s, the traditional EMAC scheme failed to compute the channel flow. On the other hand, the proposed IMAC scheme was able to simulate very well even in the case of $\Delta t = 100$ s. All the relative errors for the $\Delta t$ (s) values varying from 1 to 100 are less than 0.5%, which demonstrates once more that the proposed IMAC model is accurate, robust and highly stable.

To avoid the accumulation of the initial error caused by using a very large initial time step, a linear increase in the value of $\Delta t$ was employed for the higher time increment values. The CPU times required for the EMAC scheme ($\Delta t = 1$ s), the IMAC scheme ($\Delta t = 10$ s) and the IMAC scheme ($\Delta t = 100$ s) are 22.892 s, 6.1602 s and 1.1211 s, respectively. As shown in Table 3, the values of the *Efficiency* index for the EMAC scheme ($\Delta t = 1$ s), the IMAC scheme ($\Delta t = 10$ s) and the IMAC scheme ($\Delta t = 100$ s) are 1.00, 3.72 and 20.42, respectively. The results evidently prove that the proposed IMAC scheme, which allowed the use of time steps of magnitudes bigger than those in the EMAC scheme, is a very robust algorithm which can be employed to calculate the transient flood wave in steep rivers.

Table 3 also lists the simulation results for the channel flow problem using the INKW scheme. It is shown that the efficiency is nearly the same between the IMAC and INKW

**Table 3** | Simulation results for the channel flow problem

| Schemes | $\Delta x$ (m) | $\Delta t$ (s) | $L_2^m$(%) Depth ($y$) | $L_2^m$(%) Discharge ($Q$) | CPU time (s) | Efficiency |
|---|---|---|---|---|---|---|
| EMAC | 30 | 1 | $2.35 \times 10^{-3}$ | $3.88 \times 10^{-3}$ | 22.892 | 1.00 |
| IMAC | 30 | 1 | $2.35 \times 10^{-3}$ | $3.88 \times 10^{-3}$ | 23.305 | 0.98 |
| EMAC | 30 | 2 | N/A | N/A | N/A | N/A |
| IMAC | 30 | 2 | $2.81 \times 10^{-3}$ | $4.64 \times 10^{-3}$ | 16.734 | 1.37 |
| IMAC | 30 | 5 | $3.99 \times 10^{-3}$ | $6.59 \times 10^{-3}$ | 9.4141 | 2.43 |
| IMAC | 30 | 10 | $6.12 \times 10^{-3}$ | 0.0101 | 6.1602 | 3.72 |
| IMAC | 30 | 50 | 0.0309 | 0.0512 | 1.9922 | 11.49 |
| IMAC | 30 | 100 | 0.0717 | 0.1192 | 1.1211 | 20.42 |
| INKW | 30 | 1 | 0.0101 | $4.89 \times 10^{-3}$ | 80.467 | 0.28 |
| INKW | 30 | 2 | 0.0143 | $7.30 \times 10^{-3}$ | 39.848 | 0.57 |
| INKW | 30 | 5 | 0.0230 | 0.0137 | 17.004 | 1.35 |
| INKW | 30 | 10 | 0.0335 | 0.0245 | 9.0625 | 2.53 |
| INKW | 30 | 50 | 0.0994 | 0.1303 | 1.7422 | 13.14 |
| INKW | 30 | 100 | 0.1880 | 0.2883 | 0.9414 | 24.32 |

*Note*: "N/A" = Not Available.

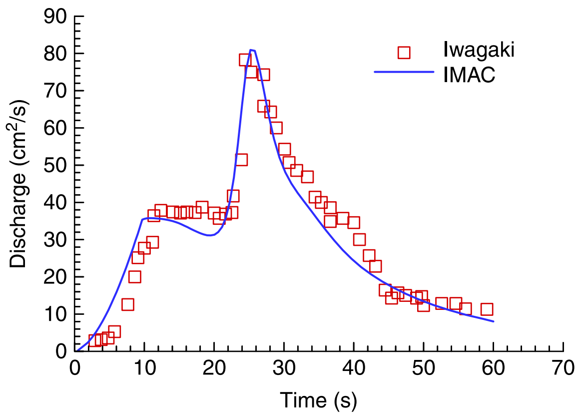


**Figure 6** | Discharge hydrograph for flow over three-plane cascade with rainfall duration of 10 s.

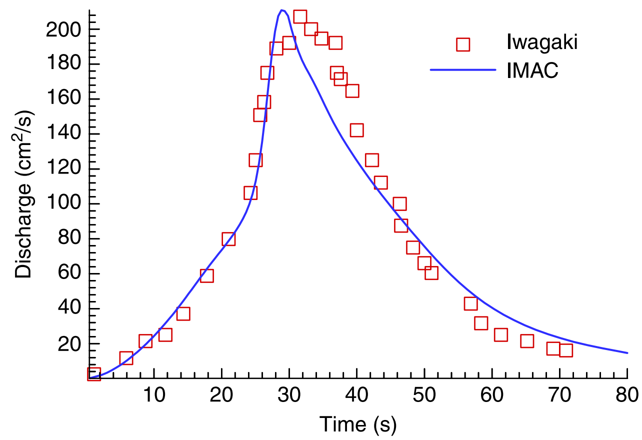


**Figure 8** | Discharge hydrograph for flow over three-plane cascade with rainfall duration of 30 s.

schemes. However, the accuracy is almost a magnitude lower than that in the IMAC scheme. Since the INKW scheme only has first-order accuracy, the computation accuracy is significantly less than that of the second-order scheme (IMAC).

### Overland flow over a three-plane cascade surface

The above test cases only compared the simulation with analytical solutions and results of other schemes. To illustrate that the proposed model is capable of describing real rainfall–runoff scenarios, the model prediction is compared with the laboratory experiments of Iwagaki (1955). The experiments were conducted to simulate rainfall–runoff processes over a three-plane cascade surface. A series of

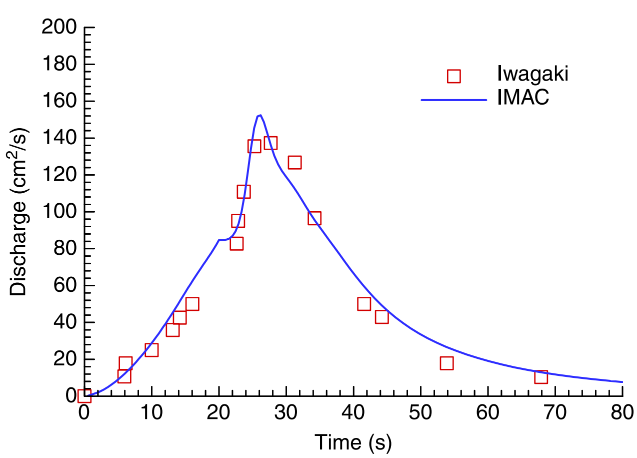


**Figure 7** | Discharge hydrograph for flow over three-plane cascade with rainfall duration of 20 s.

artificial rainfall was applied on a smooth aluminum laboratory flume surface. The flume is 24 m in length and is divided into three planes with constant slopes of 0.020, 0.015 and 0.010, respectively, from upstream to downstream. The constant rainfall intensities of 389, 230 and 288 cm/h were applied to the upper, middle and lower plane, respectively, for durations of 10 s, 20 s and 30 s in three experiments.

The flow domain is discretized into 97 grids with a uniform grid spacing $\Delta x = 0.25$ m and the Courant number CFL $= 5.0$. Manning coefficients of 0.085, 0.011 and 0.013 were employed for the three test simulations, respectively. The computed outflow hydrographs compared with experimental data for the three cases are shown in Figures 6–8. The agreement between the prediction and the experimental results is satisfactory. This result compares favorably with those of Borah *et al.* (1980) with a kinematic shock-fitting model, Zhang & Cundy (1989) with the explicit MacCormack finite-difference scheme and Tisdale *et al.* (1998) with the upwind finite element method. Also, it is shown that the proposed IMAC scheme is capable of simulating the kinematic shock waves which were generated by the changes in bed slopes for the cases of short rainfall duration.

## CONCLUSIONS

A simple, accurate and efficient kinematic wave numerical model is developed to simulate the one-dimensional overland and open-channel flows by using a family of MacCormack-type finite-difference schemes, namely the explicit MacCormack (EMAC) scheme and the implicit MacCormack (IMAC) scheme. The accuracy, efficiency and robustness of the proposed model were verified with two analytical solutions and one experimental measurement for transient overland and open-channel flows. The verifications produced relevant data agreements. These simulated results demonstrated that the proposed IMAC model exhibited high accuracy and robust stability.

Comparisons with the traditional EMAC model showed that a new feature has been enhanced in the proposed model. Based on a direct extension of the EMAC scheme, an implicit correction step was added to offset the restriction of the usual CFL stability condition. Furthermore, the computation efficiencies of the IMAC scheme increased approximately 20 times faster than those of the EMAC scheme for both the overland and channel flows. In comparison with the implicit nonlinear kinematic wave (INKW) scheme, it was shown that the INKW scheme seemed slightly more efficient than the IMAC scheme. However, the accuracy of the former appeared almost a magnitude lower than that of the IMAC scheme.

Based on the results of this study, the IMAC scheme is the best choice for the simulation of kinematic waves flow in overland and open-channel cases. This scheme can allow the use of time steps of larger magnitudes than those in the explicit schemes. Likewise, the scheme achieved a second-order accuracy in space and time. In addition, the IMAC scheme can remain stable and accurate, provided that the CFL does not become too large to prevent the calculation of the time-dependent problems. In other words, the IMAC scheme has a bidiagonal nature that could allow the explicit evaluation of additional implicit corrections. In short, the IMAC scheme does not require a lot of computational effort. This contributes to the simplicity, accuracy and efficiency of the proposed model.

## ACKNOWLEDGEMENTS

A portion of this work was supported by the National Science Council, ROC, under grant no. NSC-97-2211-E-040-007. This support is greatly appreciated.

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# 國科會補助專題研究計畫項下出席國際學術會議心得報告

日期：99 年 09 月 28 日

| | | | |
|---|---|---|---|
| 計畫編號 | NSC 98 － 2211 － E － 040 － 011 | | |
| 計畫名稱 | 應用空間資訊技術與混合數值算則於 DEM-格網式洪流演算模式之研發(1) | | |
| 出國人員姓名 | 曾明性 | 服務機構及職稱 | 中山醫學大學應資系教授 |
| 會議時間 | 99 年 7 月 11 日至<br>99 年 7 月 14 日 | 會議地點 | 青島，山東，中國 |
| 會議名稱 | 2010 International Conference on Machine Learning and Cybernetics | | |
| 發表論文題目 | A study on cluster validity using intelligent evolutionary K-MEANS approach | | |

一、參加會議經過

International Conference on Machine Learning and Cybernetics (ICMLC)為「IEEE-Machine Learning and Cybernetics」領域的重要研討會，每年舉辦一次，會議目的是提供一個國際性的傳播論壇，討論關於機器學習和控制論最新相關的革新、理論和應用的訊息。也將促進合作和共同努力，使會議的每一個參加者均能共同推動理論與實踐，以確定機器學習及控制論的主要趨勢。自2002年開辦至今ICMLC會議都非常成功，今年投稿論文審稿機制更是嚴格，只要有違反學術倫理如抄襲或段落雷同都會被ICMLC的論文比對系統追出來。今年會議在青島海爾洲際飯店舉行，會議日程包括 plenary talks，tutorials，PhD colloquia，panel discussion，oral presentation and poster，是一次非常高標準的國際會議。

本次的會議為期共四天 (2010/07/11~14)，會議的第一天為兩場tutorials的訓練課程："How to disseminate your research results: essentials of effective publishing" and "Multiple Classifier Systems"。第二天上午是兩場邀約演講，而後從下午開始則是各個session的論文發表行程，所有的作者一一上台報告今年最新發表的論文；而在其中間則是會有PhD colloquia panel talks，best paper award and best student conference paper等議程，讓世界各國的學者都可以參與主題的討論，進行學術上的交流。

本人被安排在 12 日下午的 session: intelligent systems IV，發表論文題目為「A study on cluster validity using intelligent evolutionary k-MEANS approach」。

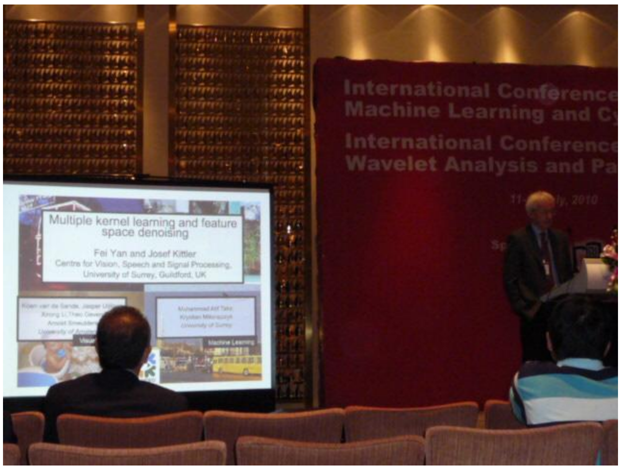
二、與會心得

本次會議研討內容涵蓋範圍很廣包括：Adaptive systems, Neural net and support vector machine, Business intelligence, Hybrid and nonlinear system, Biometrics, Fuzzy set theory, fuzzy control and system, Bioinformatics, Knowledge management, Data and web mining, Information retrieval, Intelligent agent, Intelligent and knowledge based system, Financial engineering, Rough and fuzzy rough set, Inductive learning, Networking and information security, Geoinformatics, Evolutionary computation, Pattern recognition, Ensemble method, Logistics Information fusion, Intelligent control, Visual information processing, Media computing, Computational life science.

這是本人第一次參與國外舉辦的國際性學術研討會，除了見識到國際性大型研討會的規模外並深刻感受到學術交流所蘊含的價值，參與國際性研討會可以藉由參加許多場次的演講以及討論增廣各知識領域的範疇。於個別 session 討論時，雖然每個作者的報告只有約 15 分鐘，不足以讓人對報告者之研究有充分的了解，但卻可以讓與會者對其研究有初步的認識，足以讓個人對現今國際上有關 Machine Learning and Cybernetics 領域相關研究的趨勢及方法都能有更宏觀的認識，這對個人往後的研究乃有莫大的助益，而主辦單位於各個相關主題分類的安排，更可以使相同領域的研究者針對彼此不同的概念及方法做分享，藉以刺激出彼此間研究的新想法。

今年兩場邀請演講分別為 Porf. Josef Kittler，演講題目為：Multiple Kernel Learning and Feature Space Denoising、及 Prof. Bryan Scotney，演講題目為：Incompleteness in Data for Decision Making。






今年會議共有625篇論文發表(oral + poster)，作者來自許多國家，包括中國，台灣，香港，印度，伊朗，美國，英國，日本，韓國，泰國等國。由於在大陸舉辦，亞洲周邊國家的參與學者仍為多數，

約佔有8成，台灣學者也不少，足見台灣研究能力與參與國際學術的動力是相當不錯的。出席者不少為教授帶領研究生一同出席，由研究生報告居多；但本人因國科會補助經費有限，無法帶研究生一同前往實為美中不足。

總之，這次的會議，不僅見到一些久仰大名的大師與學者，彼此交換近來的經驗與資訊，對於未來研究增長許多見聞；同時，與許多素未謀面的學術同好互相切磋觀摩而結良友，大家對於未來國際合作多持正面樂觀的態度，對於提升本校(系)以及台灣的學術地位均有相當大的助益。

## 三、考察參觀活動(無是項活動者略)

本人趁著這次出國到大陸開會的機會，自費回程順道轉機前往北京，親身體會大陸一流學府：北京大學與清華大學的不同風貌。雖然適逢暑假期間，但校園都還是人潮擁擠，北京大學校舍均屬中西合壁形式構築，而清華大學則一致呈現西方建築風。


## 四、建議

本此參加ICMLC2010國際會議，深感參與會議能增加國際能見度，可達到提升台灣的學術地位的重要。另外，各國學者均有以國際研討會來訓練研究生的趨勢，多數的論文發表均為教授帶領研究生一同出席，可提高研究品質，建立研究生的研究自信，並擴展國際視野。若未來國科會計畫也能提供研究生參與國際會議足夠的補助，對本國未來研究生的養成必有長遠的貢獻。

## 五、攜回資料名稱及內容

1. ICMLC會議論文集（全文CD及紙本書）。

2. 大會手冊，內容包括大會議程、論文發表地點、場次等。

無研發成果推廣資料

# 98 年度專題研究計畫研究成果彙整表

| 計畫主持人：曾明性 | | | 計畫編號：98-2221-E-040-011- | | | | |
|---|---|---|---|---|---|---|---|
| 計畫名稱：應用空間資訊技術與混合數值算則於 DEM-格網式洪流演算模式之研發 | | | | | | | |
| 成果項目 | | | 量化 | | | 單位 | 備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等） |
| | | | 實際已達成數（被接受或已發表） | 預期總達成數(含實際已達成數) | 本計畫實際貢獻百分比 | | |
| 國內 | 論文著作 | 期刊論文 | 0 | 0 | 100% | 篇 | |
| | | 研究報告/技術報告 | 0 | 0 | 100% | | |
| | | 研討會論文 | 0 | 0 | 100% | | |
| | | 專書 | 0 | 0 | 100% | | |
| | 專利 | 申請中件數 | 0 | 0 | 100% | 件 | |
| | | 已獲得件數 | 0 | 0 | 100% | | |
| | 技術移轉 | 件數 | 0 | 0 | 100% | 件 | |
| | | 權利金 | 0 | 0 | 100% | 千元 | |
| | 參與計畫人力（本國籍） | 碩士生 | 1 | 1 | 100% | 人次 | |
| | | 博士生 | 0 | 0 | 100% | | |
| | | 博士後研究員 | 0 | 0 | 100% | | |
| | | 專任助理 | 0 | 0 | 100% | | |
| 國外 | 論文著作 | 期刊論文 | 1 | 1 | 100% | 篇 | |
| | | 研究報告/技術報告 | 0 | 0 | 100% | | |
| | | 研討會論文 | 0 | 0 | 100% | | |
| | | 專書 | 0 | 0 | 100% | 章/本 | |
| | 專利 | 申請中件數 | 0 | 0 | 100% | 件 | |
| | | 已獲得件數 | 0 | 0 | 100% | | |
| | 技術移轉 | 件數 | 0 | 0 | 100% | 件 | |
| | | 權利金 | 0 | 0 | 100% | 千元 | |
| | 參與計畫人力（外國籍） | 碩士生 | 0 | 0 | 100% | 人次 | |
| | | 博士生 | 0 | 0 | 100% | | |
| | | 博士後研究員 | 0 | 0 | 100% | | |
| | | 專任助理 | 0 | 0 | 100% | | |

| | |
|---|---|
| 其他成果<br>(無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。) | 本研究屬第一年計畫，全期三年研究主軸在應用空間資訊技術與混合&amp；#63849；值算則進行集水區分佈型洪水演算模式之開發，本年度著重在運動波演算模式及擴散波演算模式之開發。目前已發表一篇論文在 Journal of Hydroinformatics, 採用隱式 MacCormack 算則進行運動波演算模式的開發與驗證。在規劃水文、水資源問題上常面&amp；#63990；規劃地點無適當&amp；#63870；測資&amp；#63934；可供分析，因此發展基於水力演算之物理分佈型洪流演算模式，對於土地利用發生變化或無水文紀錄地區之降雨逕流推估與積水深度計算，其適用性有相當大的貢獻。 |

| | 成果項目 | 量化 | 名稱或內容性質簡述 |
|---|---|---|---|
| 科教處計畫加填項目 | 測驗工具(含質性與量性) | 0 | |
| | 課程/模組 | 0 | |
| | 電腦及網路系統或工具 | 0 | |
| | 教材 | 0 | |
| | 舉辦之活動/競賽 | 0 | |
| | 研討會/工作坊 | 0 | |
| | 電子報、網站 | 0 | |
| | 計畫成果推廣之參與（閱聽）人數 | 0 | |

# 國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

　　□實驗失敗

　　□因故實驗中斷

　　□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

Tseng, M. H. (2010). ’Kinematic wave computation using an efficient implicit method’, Journal of Hydroinformatics, 12(3), 329-338. (SCI)

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

本研究屬第一年計畫，全期三年研究主軸在應用空間資訊技術與混合&amp；#63849；值算則進行集水區分佈型洪水演算模式之開發，本年度著重在運動波演算模式及擴散波演算模式之開發。目前已發表一篇論文在 Journal of Hydroinformatics, 採用隱式 MacCormack 算則進行運動波演算模式的開發與驗證。在規劃水文、水資源問題上常面&amp；#63990；規劃地點無適當&amp；#63870；測資&amp；#63934；可供分析，因此發展基於水力演算之物理分佈型洪流演算模式，對於土地利用發生變化或無水文紀錄地區之降雨逕流推估與積水深度計算，其適用性有相當大的貢獻。